The Dynamical Behavior of a Single-Mode Optical Fiber Strain Gage

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Abstract—**A comparison is reported between the dynamical response of a single-mode fiber optic and resistive strain gages in the frequency range 25–250 Hz. The integral phase change of the laser light propagation into the vibrating fiber optic is evaluated. A mechanical system was designed to exactly delimitate the part of the optical fiber which is subjected to vibration. The frequency spectrum of the phase change signal and the resistive strain gage signal show equivalent behavior. This result is in good agreement with the theoretically expected values and confirms the validity of the analysis performed.**

I. Introduction

In recent years, great effort has been devoted to the development of fiber-optic sensors [1] especially acoustic detectors [2]. One of the other sensors has been the optical fiber strain gage demonstrated by Butter and Hocker [3]. They presented experimental results obtained in a static configuration, that is the case when the fiber was stretched in a continuous mode and the interferential fringe shifts were counted.

In this paper we extend the above considerations to the experimental situation in which the dynamical behavior of a fiber-optic strain gage was illustrated, and present related theoretical analysis. A comparison with a conventional resistive strain gage is also reported. Optical fiber sensors can be very useful as dynamical strain gages because they have no practical limitations on the mechanical frequency range and measurement lengths and because they have an inherent immunity from electrical disturbances in signal transmission.

II. Experimental Setup

A. Mechanical System

For the fiber-optic strain gage, the response is directly proportional to the length of the fiber subjected to deformation. Since the lead fiber can also be subjected to a strain field causing a potential calibration error, the first problem to overcome in the dynamical evaluation of the fiber strain sensor is to isolate exactly the part of the fiber subjected to vibration. One should note, incidentally, that such a problem has not been met in acoustic-fiber sensors due to the particular configuration of the device in which many meters of fiber are wrapped on a coil to obtain maximum sensitivity to the acoustic wave.

Fig. 1 shows the realized mechanical system. The single-mode optical fiber is cemented on a thin bar 1 mm thick, 10 mm large, and 114 mm long, and hinged to massive supports. To best approximate the theoretical hinged-hinged constrain, two pivots are soldered on the bar ends, and subsequently, fit in hard metal "V" housing. In this mode, the vibration of the bar causes only a pivot rotation and not any displacement of the bar ends. Then, because of the extremely small rotations involved in our measurements, the optical fiber subjected to vibration is well insulated from the remaining part. The fiber is cemented along the whole length of the bar beside the resistive strain gage. The ends of the fiber are rigidly fixed to separate massive supports.

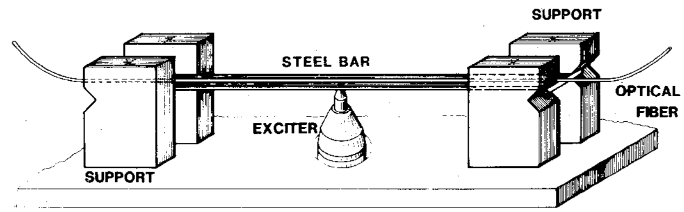

Fig. 1. Sketch of the adopted mechanical mounting. Shown is the steel bar, the hinged contraints, the optical fiber cemented on the bar, and the exciter.

In order to compare the dynamical response of a conventional strain gage with a fiber-optic strain gage, a fiber with the same length as a conventional strain gage was used and excited.

The chosen strain gage, the longest of the Micro-Measurements catalogue model 40 CBY guarantees that the whole bar was almost covered. The active gage length is nearly the same as the fiber-active length. The strain gage and the optical fiber were cemented on the bar by means of the same adhesive, a two-component epoxy MM AE–12 recommended for the dynamical stress analysis. The bar was excited exactly in the middle point by a minishaker B e K Model 4810. At this point, the vibration amplitude was monitored by a proximity sensor, Bentley and Nevada Model 7200.

B. Optical Setup

When an optical fiber is subjected to mechanical vibration, the resulting variations in length, diameter, and refractive index cause the optical path to change. The induced phase change in the laser beam passing through the fiber can be detected by an interferometric setup. We have adopted a classical Mach–Zehnder scheme where one arm is the optical fiber subjected to dynamical strain as shown in Fig. 2. The other arm goes through an ADP crystal. The chosen detection arrangement, a differential configuration, guarantees a linear output for small phase shifts around a $\pi/2$ phase difference and allows a high common-mode noise rejection [4]. The ADP crystal, acting as phase modulator is inserted into a feed-

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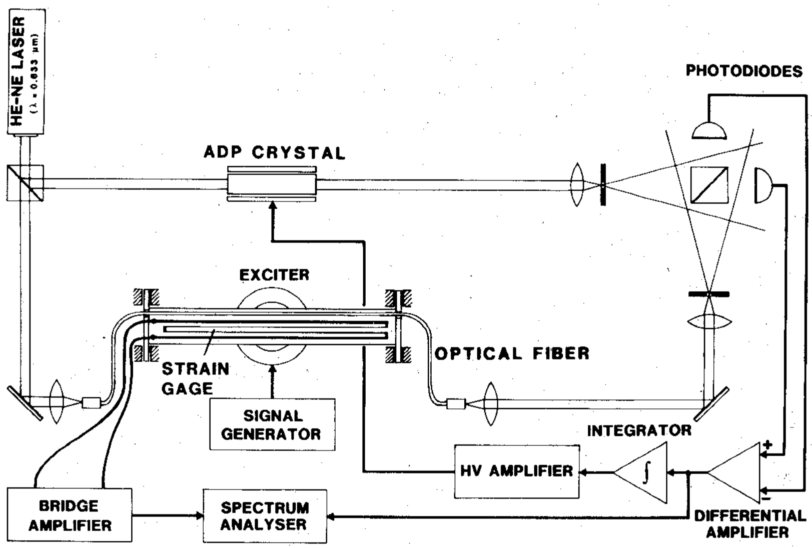

Fig. 2. Setup of the Mach-Zehnder interferometer. Shown is the slow phase-shift compensation loop which drives the ADP crystal.

back loop with a long time constant. In this way the compensation of the slow phase drifts, caused mostly by thermal effects, is achieved. Moreover, the $\pi/2$ locking point gives the maximum sensitivity for the detection scheme.

The He-Ne laser beam ($\lambda = 0.633$ μm) is divided by a beam splitting cube and coupled into the single-mode fiber (Valtec SM05) by means of a single He-Ne corrected best-form lens. The beams from the fiber and the ADP are collimated into spatial filters and the resulting spherical wavefronts are superimposed, by means of a second beam splitter, to produce the interference fringes.

III. Analysis

A. Mechanical Analysis

In order to theoretically evaluate the phase change in the fiber-optic strain gage, we need to be able to describe fully the strain state of the bar during the vibration. In fact, it is reasonable to suppose that the fiber cemented on the bar is submitted to the same deformation state.

A hinged-hinged bar forced to vibration with a force directed along the z axis and acting in its central point is subjected to a moment $M(x, t)$ and to its balancing shearing force $Q(x, t) = dM(x, t)/dx$, where x is the coordinate along the bar. From this stress state follows [5] the deformation state which can be represented, in a contracted notation, as the following:

$$S_j(x, t) = \begin{bmatrix} \epsilon_{xx}(x, t) \\ -\mu\epsilon_{xx}(x, t) \\ -\mu\epsilon_{xx}(x, t) \\ 0 \\ \epsilon_{xz}(x, t) \\ 0 \end{bmatrix} \tag{1}$$

where μ is the Poisson's ratio and $\epsilon_{xx}(x, t)$, $\epsilon_{xz}(x, t)$ are the space- and time-dependent strain components. Their values can be obtained from the vibrational analysis of the hinged-hinged bar. In a general way, the vibrational state of a system may be expressed as superposition of the principal modes of motion. As the frequency range chosen for the measurements 20–250 Hz include only the first natural mode of vibration, our analysis is simplified.

Let $u(L/2, \nu)$ be the displacement value in the middle point of the vibrating bar of length L at frequency ν, then the time-dependent conduct of the displacement of the bar in the z direction at the coordinate x and frequency ν is given by [6]

$$s(x, t) = u\left(\frac{L}{2}, \nu\right) \sin \frac{\pi}{L} x \sin 2\pi\nu t. \tag{2}$$

The value of $u(L/2, \nu)$ depends on ν around the resonant frequency ν_r with a classical resonance behavior. From the expression (2) it is simple to derive the expressions of $M(x, t)$ and $Q(x, t)$, and consequently, of $\epsilon_{xx}(x, t)$, $\epsilon_{xz}(x, t)$. In particular, the expression of ϵ_{xx} in terms of spectral component of $u(L/2, \nu)$ will be

$$\epsilon_{xx}(x, \nu) = a \frac{\pi^2}{L^2} u\left(\frac{L}{2}, \nu\right) \sin \frac{\pi}{L} x \tag{3}$$

where a is the semithickness of the bar.

B. Fiber-Optic Strain Gage

To obtain the overall phase modulation $\Delta\phi(\nu)$ of the light field propagation through the vibrating fiber, it is necessary to integrate the contribution of each infinitesimal fiber element for the whole active length l of the fiber [7]

$$\Delta\phi(\nu) = \int_0^l \frac{d\phi(x, \nu)}{dx} dx. \tag{4}$$

The induced differential phase change $d\phi$ may be written as

$$d\phi = \boldsymbol{\beta} \cdot dl + l \cdot d\boldsymbol{\beta} \tag{5}$$

where dl is the differential elongation of the fiber and $d\beta$ the differential change of the single-mode propagation vector β.

Two effects may contribute to $d\beta$, that is, the strain-optic effect and the waveguide mode-dispersion effect. The second effect may be considered negligible with respect to the first one. Since the core-cladding refractive index difference is very small, we can write for z-polarized light

$$l\, d\beta = -\frac{1}{2}\, l\, \beta_o n^3\, d\left(\frac{1}{n^2}\right)_z. \tag{6}$$

In this expression β_o represents the free space propagation wave vector equal to $2\pi/\lambda_o$ and $d(1/n^2)$ is the differential change of the optical indicatrix in the light polarization direction.

Now, considering dl as the differential elongation component in the light propagation direction, the expression (5) becomes

$$d\phi = \beta dl - \frac{1}{2}\, l\beta_o n^3\, d\left(\frac{1}{n^2}\right)_z. \tag{7}$$

The knowledge of the strain state of the fiber permits us to compute the two differential terms in the second member.

The differential elongation component is

$$dl = \epsilon_{xx}\, dx \tag{8}$$

and the differential change of the optical indicatrix $d(1/n^2)$ can be evaluated as follows. Let us consider the strain-optic tensor p_{ij} for an isotropic homogeneous material [8]

$$p_{ij} = \begin{bmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{bmatrix}. \tag{9}$$

The index change normalized on the length l for the infinitesimal length dx is

$$d\left(\frac{1}{n^2}\right)_i = \frac{dx}{l} \sum_{j1}^{6} p_{ij} s_j \tag{10}$$

which becomes, for a z-polarized beam propagation in the x direction and by means of expression (1) of S_j

$$d\left(\frac{1}{n^2}\right)_z = \frac{1}{l}\, [p_{12} - \mu(p_{11} + p_{12})]\, \epsilon_{xx}\, dx. \tag{11}$$

By means of the expression (3) of ϵ_{xx} (x, ν) and by (7), (8), and (11), we finally obtain

$$d\phi(x, \nu) = \beta \frac{\pi^2}{L^2} a \left\{1 - \frac{n^2}{2} [p_{12} - \mu(p_{11} + p_{12})]\right\} \cdot u\left(\frac{L}{2}, \nu\right) \sin \frac{\pi}{L} x\, dx. \tag{12}$$

This expression is very important because it indicates, in a general way, the differential phase modulation induced in the laser beam emerging from the fiber cemented on the vibrating bar. The integral expression

$$\Delta\phi(\nu) = \beta \frac{\pi^2}{L^2} a \left\{1 - \frac{n^2}{2} [p_{12} - \mu(p_{11} + p_{12})]\right\} \cdot u\left(\frac{L}{2}, \nu\right) \int_0^l \sin \frac{\pi}{L} x\, dx \tag{13}$$

shows that the phase change is proportional to the vibration amplitude u of the bar and depends on the bar geometrical parameters a, L and the fiber-optic parameters n, p_{11}, p_{12}, N, l.

To compare correctly the fiber-optic strain-gage signal and the resistive strain gage, it is useful to write down explicitly the relation between the phase change $\Delta\phi$ and the so-called microstrain $\mu\epsilon$, which is the conventional unit for the strain measurements. The $\mu\epsilon$ value is defined as the ratio between the overall elongation and the active length of the strain gage, multiplied by a factor 10^6.

From (13) and from the overall elongation Δl of the fiber which is calculated as

$$\Delta l(\nu) = \frac{\pi^2}{L^2}\, a\, u\left(\frac{L}{2}, \nu\right) \int_0^l \sin \frac{\pi}{L} x\, dx \tag{14}$$

we obtain as fiber-optic microstrain at the frequency ν

$$\mu\epsilon(\nu) = 10^6 \frac{\Delta\phi(\nu)}{\beta \left\{1 - \frac{n^2}{2} [p_{12} - \mu(p_{11} + p_{12})]\right\} l}. \tag{15}$$

C. *Resistive Strain Gage*

The deformation of the bar causes on the conventional strain gage a change of its electrical resistance. This change is proportional to the overall elongation $\Delta l'$ of the strain gage with active length l'.

The $\mu\epsilon$ value of the resistive strain gage is monitored directly by means of a calibrated bridge-amplifier.

IV. Experimental Procedure and Conclusions

The shaker acting on the bar is driven by a frequency-swept signal to ensure a good mechanical excitation in the whole frequency range. The excitation power is calibrated to keep the induced phase change within the linear region of the detection arrangement. The vibrating amplitude at the point $L/2$ is below 3 μm at any frequency. The resistance variation of the strain gage is monitored by means of a special bridge-amplifier (Vishey BAM-1) battery powered and with dynamic capability. The signals from the photodiodes and from the bridge-amplifier are analyzed in real time by a fast Fourier processor (HP Model 3582 A).

As the proper mechanical excitation requires a long sweep

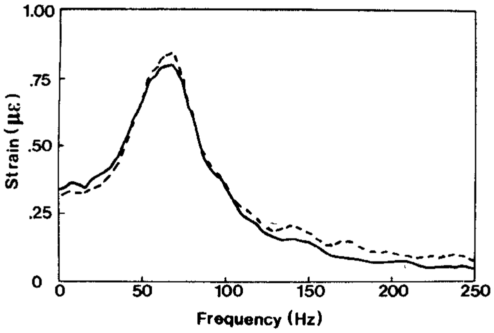

Fig. 3. Spectrum of the microstrain values obtained from the optical fiber strain sensor (full line) and from the resistive strain gage (dashed line).

time of 10 s we used for the analyzer the "peak" averaging mode, which keeps the maximum rms input at each frequency point of the spectrum. The optical setup is put on a vibration-free table and into a shield to avoid undesired acoustic noise. Fig. 3 shows the experimental results. By means of the expression (15) and the characteristic parameters reported in Table I, the phase change signal has been converted in the equivalent $\mu\epsilon$ value. The bridge-amplifier signal is calibrated in units of $\mu\epsilon$.

Plots of the rms photodiodes signal (full line) and of the rms bridge amplifier (dashed line), both obtained from a long series of sweeps, show an equivalent frequency behavior. Small differences between the two plots are principally due to the high sensitivity to noise phenomena of the peak averaging mode utilized.

The $\mu\epsilon$ values measured by the fiber-optic and resistive strain gage are also in good agreement. For example, as can be seen from the vertical scales, at the resonance frequency of 78 Hz, we have from the fiber-optic sensor a measured value of $\mu\epsilon$ = 0.82 and from the resistive strain gage a measured value of $\mu\epsilon$ = 0.86. These values confirm the correctness of the analysis which has led to the expression of $\Delta\phi$.

The comparison of the sensitivity of the two types of strain gages is one of the important aspects to be considered in the future optical sensor applications and it is worth further pondered investigation. In our experiment the residual signal from the photodetectors and the bridge amplifier, measured with the steel bar at rest, gives the average value of $2.7\ 10^{-4}$ $\mu\epsilon$ for the optical fiber strain gage and $4.1\ 10^{-3}$ $\mu\epsilon$ for the resistive strain gage. These values can be considered as indicative values of sensitivity for the two types of strain gages. The theoretical sensitivity limit for the optical fiber strain gage can be computed according to Jackson *et al.* [9]. For our detector Centronic OSI 5K and our contrast figure, the minimum detectable phase shift of the interferometer is in the order of 10^{-7} rad. This value corresponds to about 10^{-7} $\mu\epsilon$ in term of minimum detectable μ strain. The observed drop from this last and the above experimental value could be due to unwanted low-frequency acoustical noise mostly caught by the lead fiber.

TABLE I
FIBER, STEEL BAR, AND STRAIN GAGE CHARACTERISTIC PARAMETERS

STEP-INDEX OPTICAL FIBER	STRAIN GAGE	STEEL BAR
Fiber Diameter 125 10^{-6}m	Type = EA-06-40CBY-120 Micro-Measurements	Thickness 2a = 1 10^{-3}m
Core Diameter 5.38 10^{-6}m	Foil Alloy = Constantan	Width b = 10 10^{-3}m
Modulus of Elasticity 7.2 10^{10} N/m^2	Active Gage Length = 101,6 10^{-3} m	Length L = 114 10^{-3}m
Poisson's ratio μ = 0.17	Resistance = 120 Ω	Mass m = 9.47 10^{-3}kg
Mean refractive index n = 1.456		Modulus of Elasticity E = 1.57 10^{11} N/m^2
Propagation constant β = 1.446 10^7m^{-1}		Poisson's ratio μ=0.21
Photoelastic coefficient P_{11}= + 0.121		Modulus of Rigidity G = 7.73 . 10^{10} N/m^2
Photoelastic coefficient P_{12} = + 0.270		
Photoelastic coefficient P_{44}= - 0.074		
Vibrating length ℓ = 114 10^{-3} m		

In summary, we have performed an experimental evaluation between the dynamical behavior of a fiber-optic strain gage and a conventional resistive strain gage. To theoretically support this work, the integral phase change of the light propagation into a vibrating optical fiber is evaluated. The used hinged-hinged bar configuration guarantees good insulation between the part of the fiber-optic subjected to vibrations and the remaining part. The experimental results show excellent agreement between the microstrain values measured with the resistive strain gage and the fiber sensor. Besides, they confirm the correctness of the analysis developed.

The fiber sensor technology can be useful when applied to the strain dynamical measurements. It offers the classical advantages of the optical fibers, especially the electromagnetic noise immunity, in addition to a very long active length. At the present state of the technology, serious disadvantages are the acoustical and vibrational sensitivity of the lead fiber and the expensive optoelectronic signal processing. In our experiment the sensitivity of the fiber-optic strain gage is surely already comparable or better than the commonly used resistive strain gage and the theoretical limits point out the possibility to improve it.

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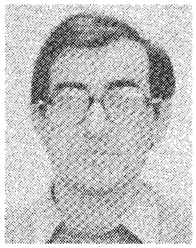

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An Electric Field Sensor Utilizing a Piezoelectric Polyvinylidene Fluoride (PVF_2) Film in a Single-Mode Fiber Interferometer

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Abstract—**A polyvinylidene fluoride (PVF_2) phase shifter is characterized in terms of amplitude response linearity, frequency response uniformity, and ultimate sensitivity to electric field. Phase-drift compensation with this PVF_2 device is demonstrated in a Mach-Zehnder fiber interferometer. The compensator can be operated at the $\pi/2$-phase mode for maximum sensitivity in detection applications, or the π-phase mode for maximum frequency mixing efficiency.**

I. Introduction

IN RECENT YEARS, research in fiber-optic sensors has demonstrated that highly sensitive devices are possible and often competitive with the best conventional devices. Areas of active research include acoustic [1], magnetic [2], temperature [3], acceleration [4], and current sensing [5]. In addition to this list, the possibility of electric field sensing should be considered. One approach would involve coating or bonding a fiber onto a piece of piezoelectric polyvinylidene fluoride (PVF_2) materials and incorporating it as a sensor element in a fiber interferometer.

Initial work [6], [7] demonstrated the feasibility of using PVF_2 film as a phase shifter and frequency mixer in an optical fiber interferometer. The attractions of PVF_2 material are that it is lightweight and possesses a large phase-shifting capability (~4 rad/V (peak)-meter at 6328 Å wavelength) as compared to that of a conventional ceramic PZT (~0.39 rad/V (peak)-meter at 6328 Å wavelength). Also, PVF_2 material has the potential of being coated directly onto optical fibers during fabrication, thus forming an integral part of the sensor.

In this paper, we present results on the characteristics of a PVF_2 phase shifter or fiber stretcher of modest length (60 cm) and its application as a lightweight, low drive-voltage, phase-drift compensator in a fiber interferometer. Important phase-shifting characteristics of the PVF_2 are the amplitude response linearity, frequency response uniformity, and the ultimate sensitivity in terms of minimal detectable electric field. Also, two operational modes of the compensator will be discussed. These include 1) the $\pi/2$-phase (quadrature) locking mode desirable for maximum sensitivity in fiber interferometric sensor

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